

Kaon physics in
the full
domain-wall QCD

Jun Noaki

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Basic parameters

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chiral extrapolation

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Summary

Kaon physics in the full domain-wall QCD

Jun Noaki

野秋淳一

RBC+UKQCD Collaborations



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Exploration of the SM with Lattice QCD

- **1st principle numerical simulation,
powerful approach to the low energy physics**

- Spectrum, decay constants ($m_\pi, m_K, m_\rho, f_\pi, f_K$ etc.)

- QCD produce hadron & PCAC relation?

- Weak matrix elements ($K^0 \leftrightarrow \overline{K^0}, K \rightarrow \pi\pi, K l \bar{l}$ etc.)

- CKM parameters?

- SM accommodates ~~CP~~ ?

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- SM accommodates ~~CP~~ ?

- **How can we do them better?**

- Chiral fermion : domain-wall QCD

- Unquenched simulation : $N_f = 2 + 1$

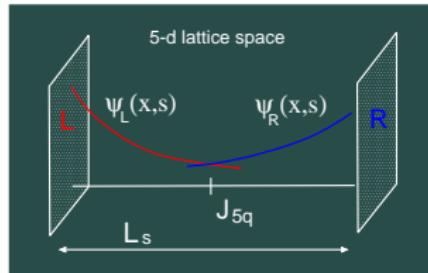
Brief history of DWQCD

1992 Domain-Wall fermion

Kaplan, Shamir

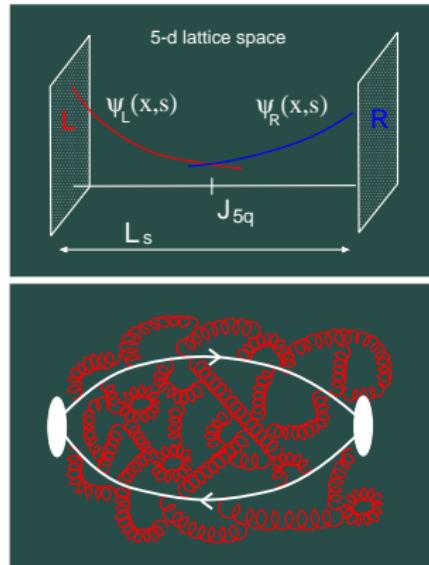
1994 Domain-Wall QCD

Fürman & Shamir



Brief history of DWQCD

- 1992 Domain-Wall fermion
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Blum & Soni, RBC
- 2000 RG improvement + DWF
CP-PACS
- 2001 $B_K, \epsilon'/\epsilon$ **CP-PACS, RBC**



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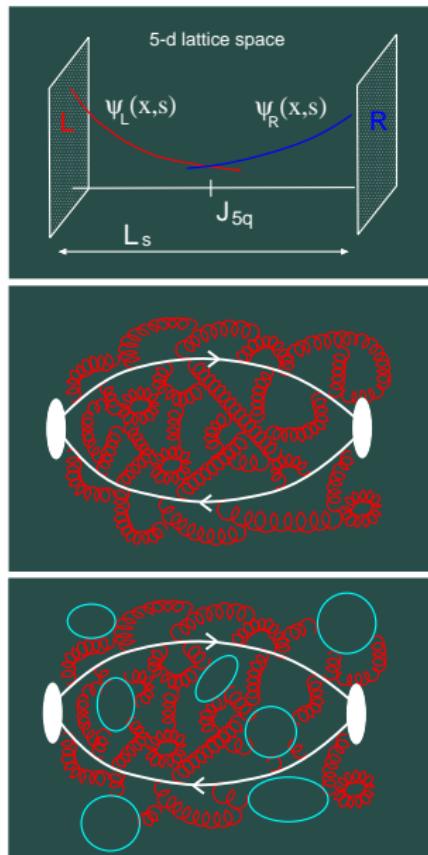
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- 2001 $B_K, \epsilon'/\epsilon$ **CP-PACS, RBC**
- 2005 $N_f = 2$ simulation
RBC
- 2006~ $N_f = 2 + 1$ simulation
RBC+UKQCD



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UKQCD + RBC Joint Collab.

● Gauge configuration

- size: **$16^3 \times 32$** (**$24^3 \times 64$** in progress)
- Iwasaki gauge action ($\beta = 2.13$)
- DWF: **$L_s = 16$, $M_5 = 1.8$**
- sea quark mass:
 $m_{u/d} = 0.01/0.02/0.03$, $m_s = 0.04$
three independent ensembles
- statistics:
 $> 300 \times 10$ trajectories for each.



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● Measurements of observables

- m_π , m_K , m_ρ , f_π , f_K , B_K
- independent calculations by UKQCD and RBC
- Narrow down the systematic error by a comparison

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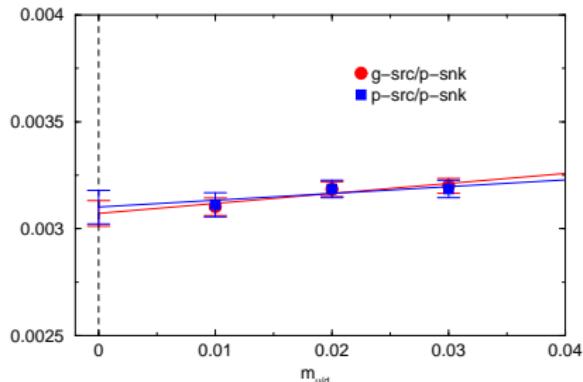
Conserved axial current

● Residual mass

Axial WTI

$$\frac{\Delta_4 \langle \mathcal{A}_4(t)P(0) \rangle}{\langle P(t)P(0) \rangle} = m_q + m_{\text{res}}$$

$$m_{\text{res}} \simeq 5 \text{ MeV}$$



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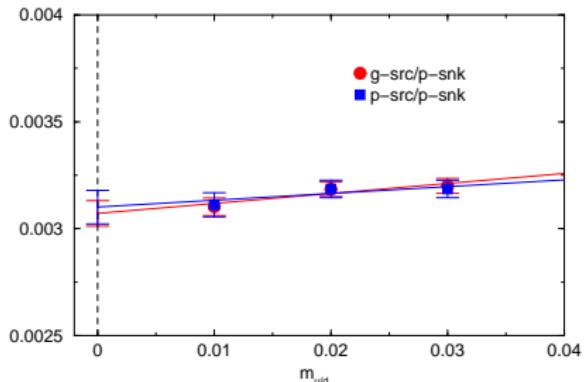
Conserved axial current

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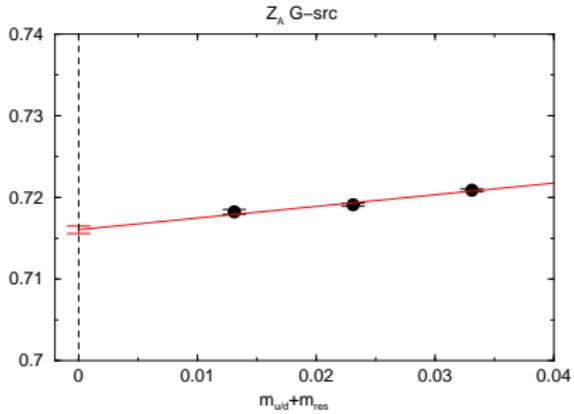
$$m_{\text{res}} \simeq 5 \text{ MeV}$$



● Renormalization factor

$$Z_A = \frac{\langle \mathcal{A}_4(t)P(0) \rangle}{A_4(t)P(0)}$$

chiral limit: $Z_A = 0.7160(4)$



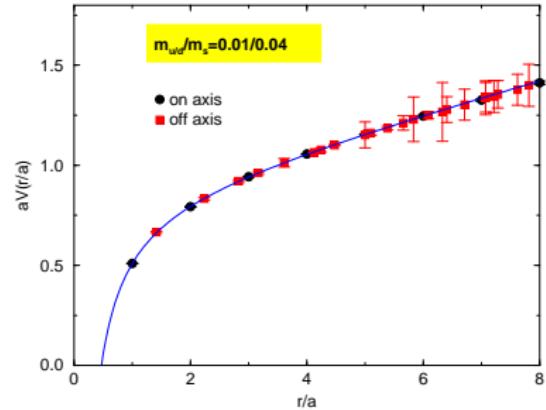
Lattice spacing

- Wilson loop & heavy quark potential

$$\langle W(r, t) \rangle = C(r) \cdot e^{-V(r)t}$$

$$V(r) = v_0 - \frac{\alpha}{r} + \sigma r$$

$$r_0^{\text{latt}} = \sqrt{\frac{1.65 - \alpha}{\sigma}}$$



Lattice spacing

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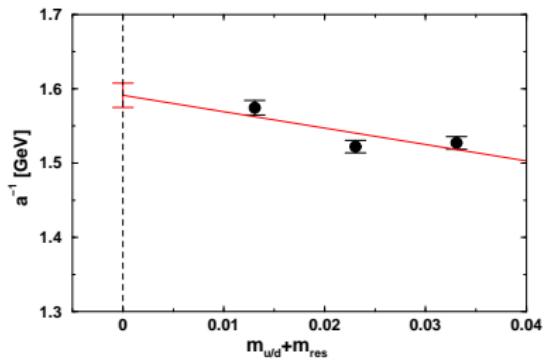
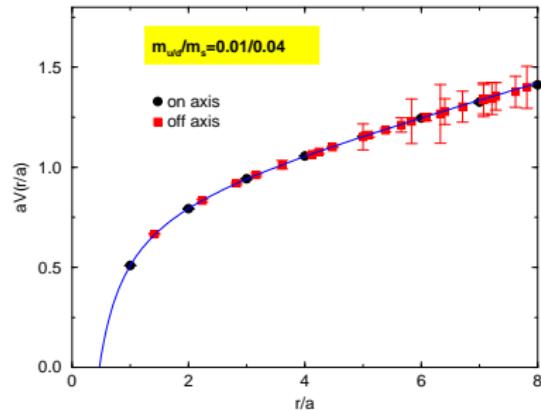
$$V(r) = v_0 - \frac{\alpha}{r} + \sigma r$$

$$r_0^{\text{latt}} = \sqrt{\frac{1.65 - \alpha}{\sigma}}$$

$$a^{-1} = r_0^{\text{latt}} / (0.5 \text{ fm})$$

chiral limit: **1.59(2)** GeV

cf. $a^{-1} = 1.56(1)$ GeV
from $m_\rho = 770$ MeV



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Spectrum & decay constants

Two-point functions

● naive mass fit

$$\langle A_4^{(\text{local})}(t) A_4^{(\text{local})}(0) \rangle = (f_\pi^{(\text{latt})})^2 m_\pi / 2 \cdot \cosh(m_\pi(T/2 - t))$$

Two-point functions

- naive mass fit

$$\langle A_4^{(\text{local})}(t) A_4^{(\text{local})}(0) \rangle = (f_\pi^{(\text{latt})})^2 m_\pi / 2 \cdot \cosh(m_\pi(T/2 - t))$$

- simultaneous fit

smeared interpolation field \Rightarrow larger overlap with the grand state

$$\left\{ \begin{array}{l} \langle A_4^{(\text{local})}(t) A_4^{(\text{smr})}(0) \rangle = f_\pi^{(\text{latt})} / 2 \cdot \langle \pi | A_4^{(\text{smr})} | 0 \rangle \cdot \cosh(m_\pi(T/2 - t)) \\ \langle A_4^{(\text{smr})}(t) A_4^{(\text{smr})}(0) \rangle = \frac{1}{2m_\pi} \left| \langle \pi | A_4^{(\text{smr})} | 0 \rangle \right|^2 \cdot \cosh(m_\pi(T/2 - t)) \end{array} \right.$$

We use Gaussian smearing.

Two-point functions

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We use Gaussian smearing.

- 9 data points (sea quarks: $m_{u/d} = 0.01/0.02/0.03$, $m_s = 0.04$)

For each ensemble, π : light-light

K : light-strange

η_s : strange-strange

Chiral extrapolation

- **Extrapolation: data points \Rightarrow physical point**
 - Chiral perturbation theory as a guide
 - No additional artifact due to DWQCD

Chiral extrapolation

- Extrapolation: data points \Rightarrow physical point

 - Chiral perturbation theory as a guide

 - No additional artifact due to DWQCD

- NLO Chiral expansion Aubin & Bernard, 2003

$$m_K^2 = \textcolor{red}{B_0}(m_s + m_d) + \xi_1(m_s + m_d)m_d + \xi_2(m_s + m_d)m_s + \alpha L_m(K),$$

$$m_\pi^2 = \textcolor{red}{B_0}2m_d + \xi_12m_s m_d + \xi_22m_d(2m_d - m_s) + \alpha L_m(\pi),$$

$$m_{\eta_s}^2 = \textcolor{red}{B_0}2m_s + \xi_12m_s(2m_d - m_s) + \xi_22m_s(3m_s - 2m_d) + \alpha L_m(\eta_s),$$

chiral-logs: $L_m(K) = \frac{4}{9}(m_s + m_d)(2m_s + m_d) \ln\left(\frac{2}{3}(2m_s + m_d)\right)$

$$L_m(\pi) = 4m_d^2 \ln(2m_d) - \frac{4}{9}m_d(2m_s + m_d) \ln\left(\frac{2}{3}(2m_s + m_d)\right)$$

$$L_m(\eta_s) = 8m_s^2 \ln(2m_s) - \frac{16}{9}m_s(2m_s + m_d) \ln\left(\frac{2}{3}(2m_s + m_d)\right)$$

Chiral extrapolation (2)

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$$\begin{aligned}f_K &= f + \beta L_f(K) + \eta_1 m_d + \eta_2 m_s \\f_\pi &= f + \beta L_f(\pi) + \eta_1 m_s + \eta_2 (2m_d - m_s) \\f_{\eta_s} &= f + \beta L_f(\eta_s) + \eta_1 (2m_d - m_s) + \eta_2 (3m_s - 2m_d),\end{aligned}$$

chiral-logs:

$$\begin{aligned}L_f(K) &= -\frac{1}{(4\pi)^2} \left[\frac{3}{2}(m_s + m_d) \ln(m_s + m_d) \right. \\&\quad \left. + (m_s + \frac{1}{2}m_d) \ln\left(\frac{2}{3}(m_s + m_d)\right) + \frac{3}{2}m_d \ln(2m_d) \right] \\L_f(\pi) &= -\frac{1}{(4\pi)^2} [4m_d \ln(2m_d) + (m_s + m_d) \ln(m_s + m_d)] \\L_f(\eta_s) &= -\frac{1}{(4\pi)^2} [2(m_s + m_d) \ln(m_s + m_d) + 2m_s \ln(2m_s)]\end{aligned}$$

- $\Lambda_\chi = 1 \text{ GeV} \Leftrightarrow m_{\eta_s} \lesssim 0.7 \text{ GeV}$
- 4 fit params from 9 data points (simultaneous fit)

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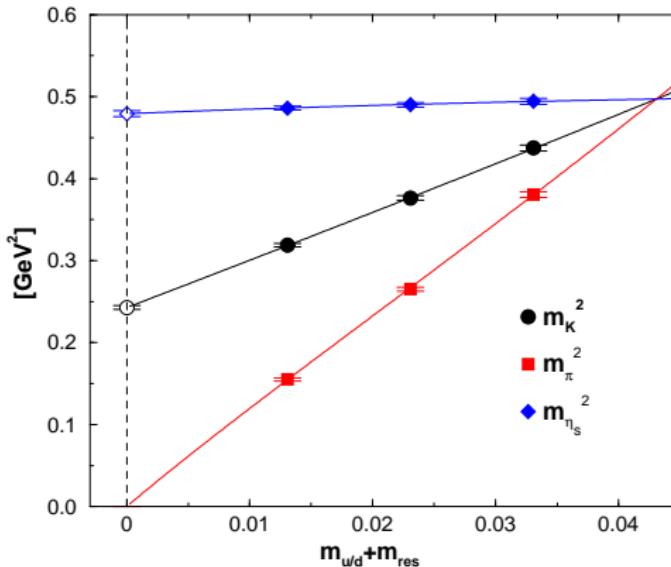
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PS meson masses



- Reproduces PCAC relation very well.
- $m_K \simeq 495$ MeV at chiral limit.
- Too few data points to check chiral property.

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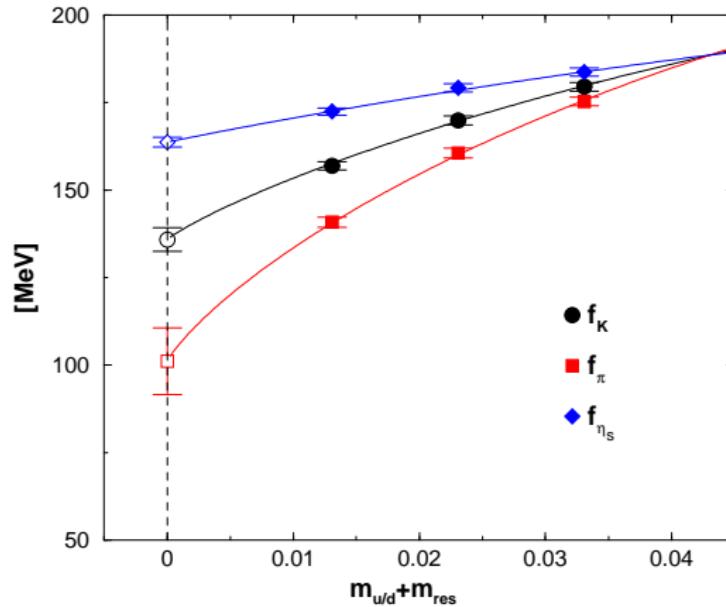
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Decay constants



- $f_K \simeq 135$ MeV, $f_\pi \simeq 101$ MeV at the chiral limit
- $f_K/f_\pi \simeq 1.31$ at the chiral limit
- B_0 is inconsistent with m_π^2 's higher order effect?

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B-parameter

Kaon B-parameter

- Parameter of $K\bar{K}$ mixing : indirect ~~CP~~ parameter

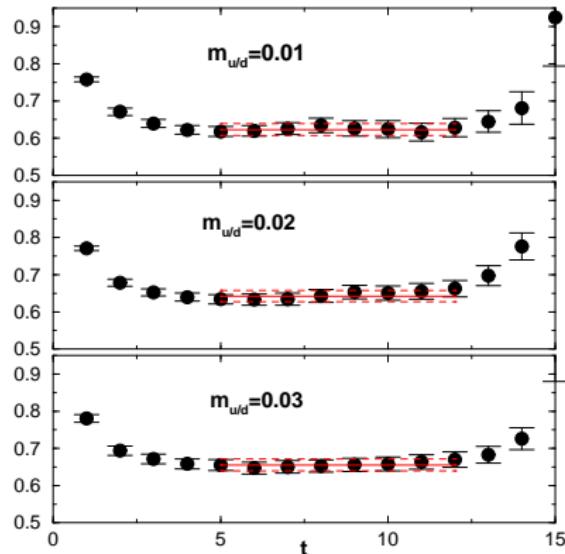
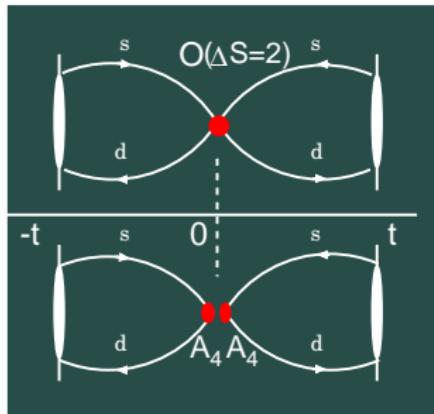
$$B_K \equiv \frac{\langle \bar{K} | O^{(\Delta S=2)} | K \rangle}{\frac{8}{3} f_K^2 m_K^2} = \frac{\langle \bar{K} | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K \rangle}{\frac{8}{3} \langle \bar{K} | A_4 | 0 \rangle \langle 0 | A_4 | K \rangle}$$

Kaon B-parameter

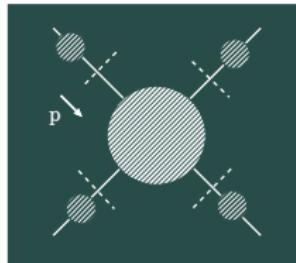
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- Plateaus



Non-perturbative Renormalization

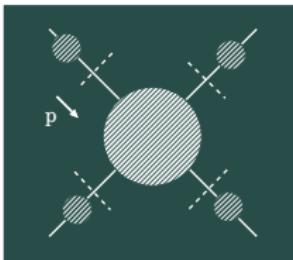


- Rome-Southampton method

$$Z_q^{-2} Z_O \Gamma_O^{(\text{latt})} = \Gamma_O^{(\text{tree})}$$

for Landau gauge (RI/MOM)

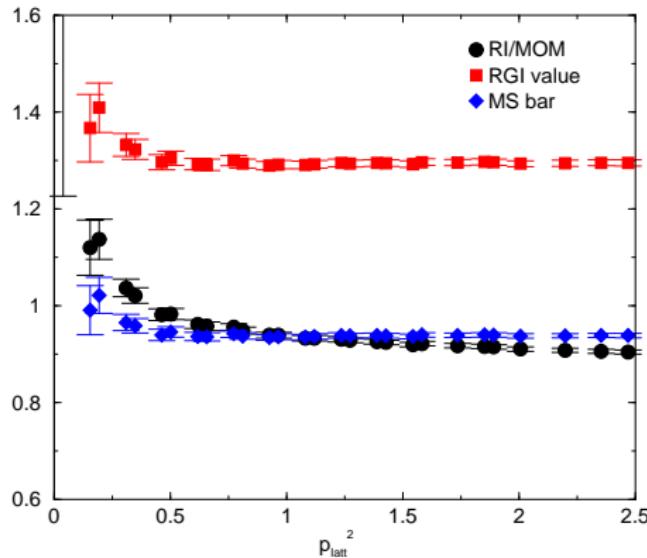
Non-perturbative Renormalization



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for Landau gauge (RI/MOM)



$$Z_{B_K}^{\text{RI/MOM}} = Z_O / Z_A^2$$

$$\begin{aligned} Z_{B_K}(\overline{\text{MS}} \text{ NDR}, \mu = 2 \text{ GeV}) \\ = 0.9382(13) \end{aligned}$$

Chiral extrapolation

• NLO Chiral expansion Van de Water & Sharpe, 2006

$$B_K = BL_B(K) + \zeta_1 m_d + \zeta_2 m_s + \zeta_3 \frac{(m_s - m_d)^2}{m_s + m_d}$$

$$B_\pi = BL_B(\pi) + \zeta_1 m_s + \zeta_2 (2m_d - m_s)$$

$$B_{\eta_s} = BL_B(\eta_s) + \zeta_1 (2m_d - m_s) + \zeta_2 (3m_s - 2m_d),$$

$$L_B(K) = 1 - \frac{B_0}{(4\pi f)^2} \left(\frac{m_d(m_s + 3m_d)}{m_s + m_d} \ln(2m_d) + 2(m_s + m_d) \ln(m_s + m_d) + \frac{(7m_s + 5m_d)(2m_s + m_d)}{3(m_s + m_d)} \ln\left(\frac{2}{3}(2m_s + m_d)\right) \right),$$

$$L_B(\pi) = 1 - \frac{6B_0}{(4\pi f)^2} 2m_d \ln(2m_d),$$

$$L_B(\eta_s) = 1 - \frac{6B_0}{(4\pi f)^2} 2m_s \ln(2m_s)$$

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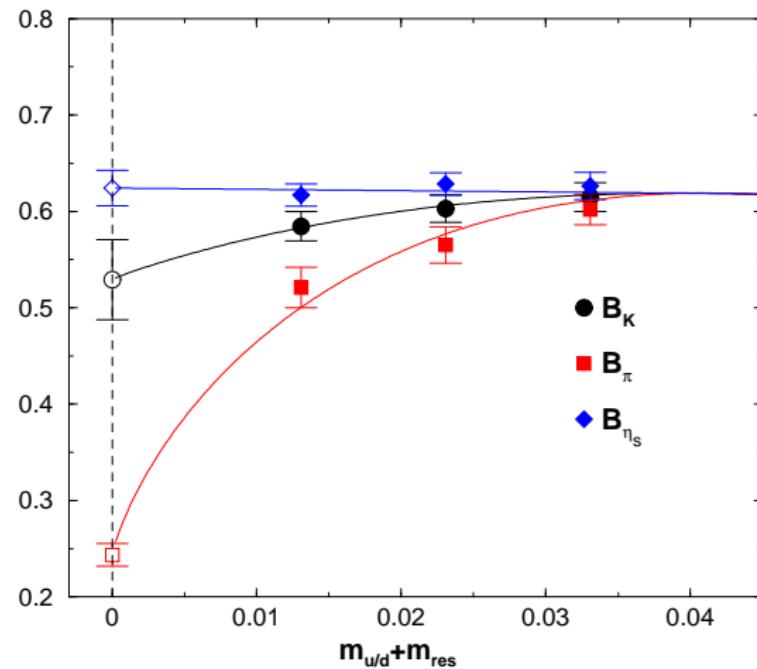
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Fit Result



$$B_K(\overline{\text{MS}} \text{ NDR}, \mu = 2 \text{ GeV}) = 0.529(42)$$

Current status of B_K

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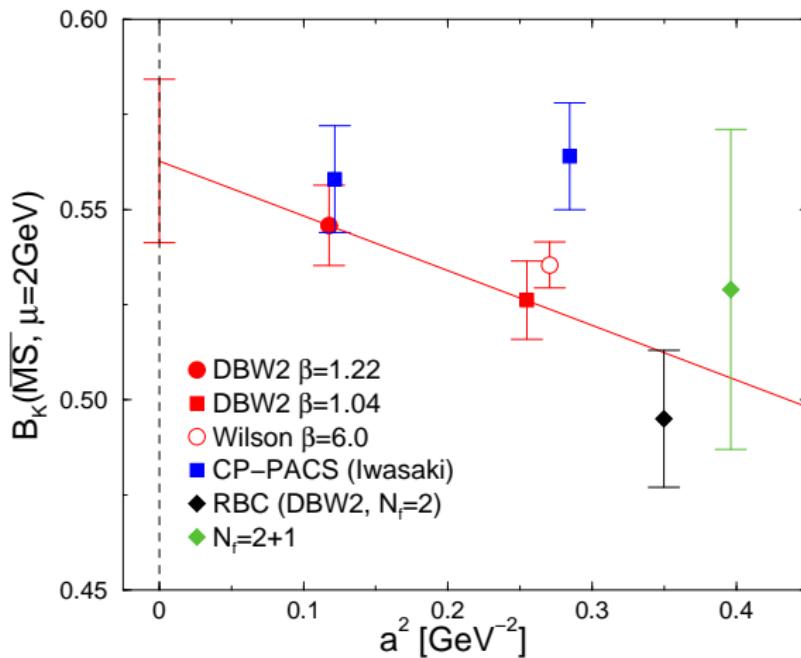
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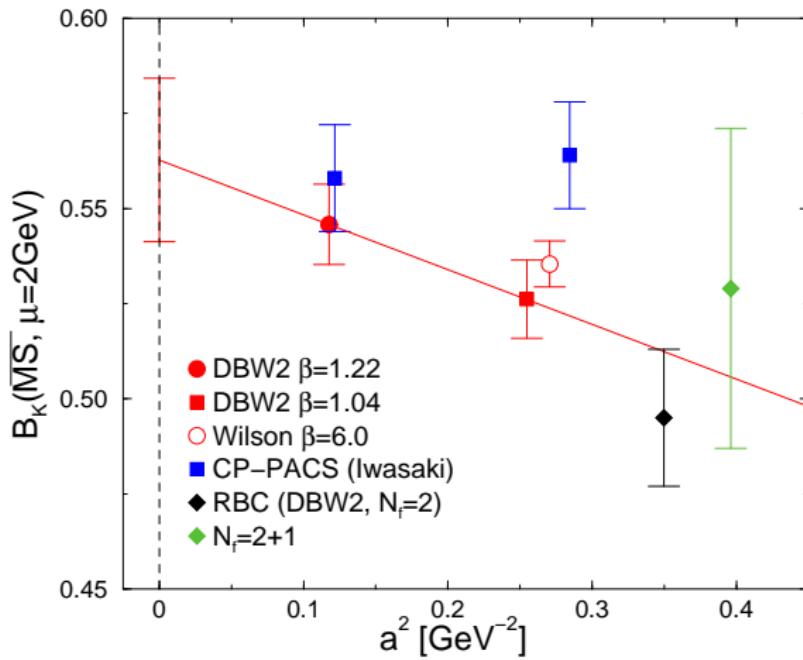
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Current status of B_K



Good starting point, isn't it?

Summary

- Preliminary physical results with $16^3 \times 32$ & $a^{-1} = 1.6$ GeV

- Phys. volume $\simeq (2 \text{ fm})^3$
- Chiral extrapolation is straightforward.
- Data look promising.

- Improvements?

- Finite size effect : $24^3 \times 64$ & $a^{-1} = 1.6$ GeV is ongoing.
- Closer to the physical points : $m_{u/d} < 0.01$ in future(?)
- Scaling violation : $a^{-1} \gtrsim 2.0$ GeV in future(?) \Rightarrow continuum limit

- Developments with many applications

- Final results from true QCD simulation are around the corner!